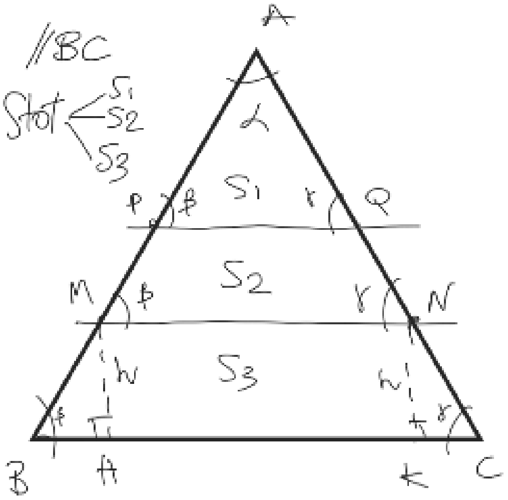
Divisione dei terreni

Divisione di un triangolo con dividenti parallele ad un lato



$APQ \simeq AMN \simeq ABC$

1) Area

$$S_1 = \frac{\overline{AP}^2 \cdot \operatorname{sen}\alpha \operatorname{sen}\beta}{2 \operatorname{sen}\gamma} \Rightarrow \overline{AP} = \sqrt{\frac{2 S_1 \operatorname{sen}\gamma}{\operatorname{sen}\alpha \operatorname{sen}\beta}}$$

2) $S_{tot} : S_1 = \overline{AB}^2 : AP^2$

$$\overline{AP} = \sqrt{\frac{S_1 \cdot AB^2}{S_{tot}}}$$

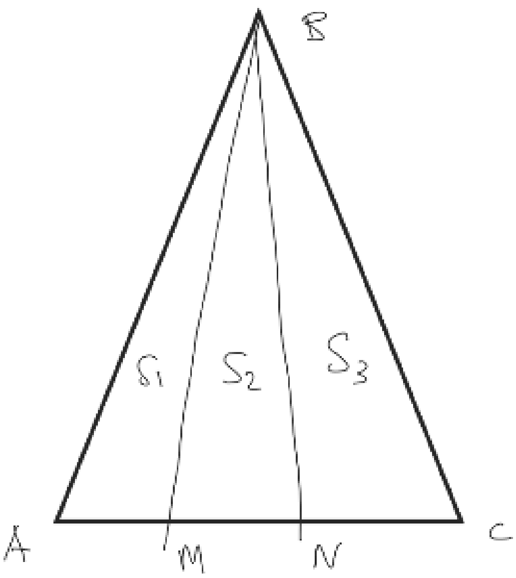
$$S_{tot} : (S_1 + S_2) = AB^2 : \overline{AM}^2$$

$$AM = \sqrt{\frac{AB^2 \cdot (S_1 + S_2)}{S_{tot}}}$$

3) formula del trapezio

$$\begin{cases} S_3 = \dfrac{\overline{BC} + \overline{MN}}{2} \cdot h \\ \overline{MN} = \overline{BC} - \overline{BH} - \overline{KC} \end{cases}$$

Divisione di un triangolo con dividente uscente da un vertice

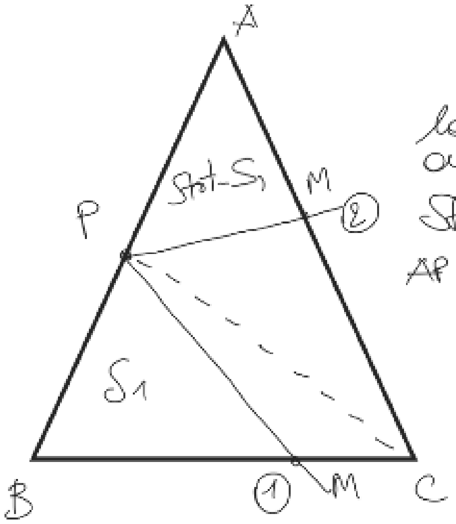


e.n
lati
angoli
Stot < S1 S2 S3

$$S_1 = S_{ABM} = \frac{\overline{AB} \cdot \overline{AM} \operatorname{sen}\alpha}{2} \Rightarrow AM = \frac{2 S_1}{\overline{AB} \operatorname{sen}\alpha}$$

$$S_1 + S_2 = \frac{\overline{AB} \cdot \overline{AN} \operatorname{sen}\alpha}{2} \Rightarrow \overline{AN} = \frac{2(S_1 + S_2)}{\overline{AB} \operatorname{sen}\alpha}$$

Divisione di un trianglo con dividenti che escono da un punto posto sul perimetro



2 n
lati
angoli
$S_{tot} < \begin{matrix} S_1 \\ S_2 \end{matrix}$
AP, BP

$$S_{PBC} = \frac{\overline{BP}\cdot\overline{BC}\,\mathrm{sen}\,\beta}{2} \gtrless S_1$$

① $S_{PBC} > S_1$

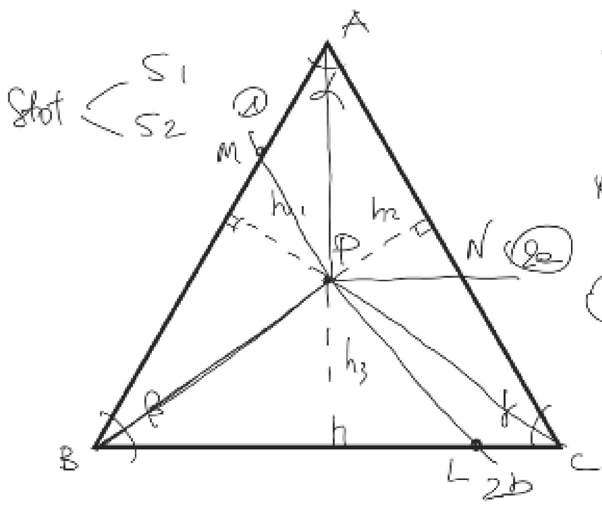
$$S_1 = \frac{\overline{BP}\cdot\overline{BM}\,\mathrm{sen}\,\beta}{2} \Rightarrow \overline{BM} = \frac{2S_1}{\overline{BP}\,\mathrm{sen}\,\beta}$$

② $S_{PBC} < S_1$

$$S_{APM} = S_{tot} - S_1 = \frac{\overline{AP}\cdot\overline{AM}\,\mathrm{sen}\,\alpha}{2}$$

$$\Rightarrow \overline{AM} = \frac{2(S_{tot}-S_1)}{AP\,\mathrm{sen}\,\alpha}$$

Divisione di un triangolo con dividenti che escono da un punto interno



$$S_{ABP} = \frac{AB\cdot h_1}{2} \gtrless S_1 \begin{cases} S_{ABP} > S_1 & ① \\ S_{ABP} < S_1 & ② \end{cases}$$

① $S_1 = S_{BMP} = \frac{\overline{BM}\cdot h_1}{2} \Rightarrow \overline{BM} = \frac{2S_1}{h_1}$

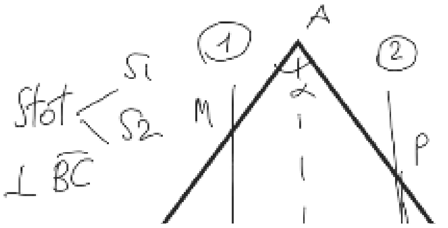
② $S_{APC} = \frac{\overline{AC}\cdot h_2}{2}$

$$S_{ABP} + S_{APC} \gtrless S_1 \begin{cases} > & 2a \\ < & 2b \end{cases}$$

2a) $S_{APN} = S_1 - S_{ABP} = \frac{\overline{AN}\cdot h_2}{2} \Rightarrow \overline{AN} = \frac{2(S_1 - S_{ABP})}{h_2}$

2b) $S_{BPL} = S_{tot} - S_1 = \frac{\overline{BL}\cdot h_3}{2} \Rightarrow \overline{BL} = \frac{2(S_{tot}-S_1)}{h_3}$

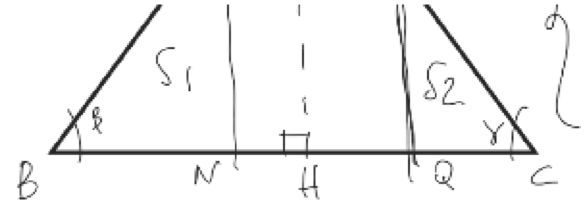
Divisione di un triangolo con dividenti perpendicolari ad un lato



$$S_{ABH} \gtrless S_1 \begin{cases} S_{ABH} > S_1 & ① \\ S_{ABH} < S_1 & ② \end{cases}$$

$$S_{ABH} = \frac{\overline{AH}\cdot\overline{BH}}{2}$$

$\ldots\ AB^2\,\mathrm{sen}\,\beta\cos\beta$



$$\begin{cases} \overline{AH} = \overline{AB} \operatorname{sen} \beta \\ \overline{BH} = \overline{AB} \cos \beta \end{cases} \Rightarrow S_{ABH} = \frac{\ldots}{2}$$

① Area $\begin{cases} S_1 = S_{BMN} = \dfrac{\overline{BN} \cdot \overline{MN}}{2} \\ \overline{MN} = \overline{BN} \cdot \operatorname{tg} \beta \end{cases}$ $\quad S_1 = \dfrac{\overline{BN}^2 \cdot \operatorname{tg}\beta}{2} \Rightarrow \overline{BN} = \sqrt{\dfrac{2 S_1}{\operatorname{tg}\beta}}$

① triangoli simili $\quad ABH \simeq BMN \quad S_{ABH} : S_1 = \overline{AB}^2 : \overline{BM}^2$

$$\Downarrow$$

$$\overline{BM} = \sqrt{\frac{S_1 \cdot AB^2}{S_{ABH}}}$$

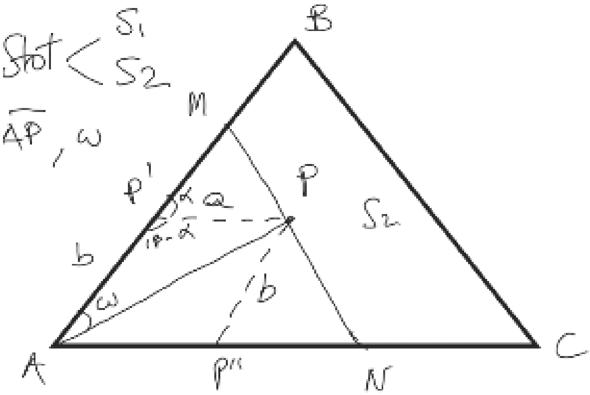
② $S_2 = S_{tot} - S_1 = S_{PQC}$

$$\overline{CQ} = \sqrt{\frac{2 S_2}{\operatorname{tg}\gamma}} \qquad \overline{CP} = \sqrt{\frac{S_2 \cdot AC^2}{S_{tot} - S_{ABH}}}$$

Divisione di un triangolo in due parti assegnate con dividenti che passa per un punto interno

$S_{tot} < \begin{matrix} S_1 \\ S_2 \end{matrix}$

$\overline{AP}, \omega$



$S_1 = S_{AMN} \quad S_2 = S_{MBCN}$

Coord. oblique $\left.\begin{matrix} PP' \parallel AC & ⓐ \\ PP'' \parallel AB & ⓑ \end{matrix}\right] \hat{APP'}$

$$\begin{cases} S_1 = \dfrac{\overline{AM} \cdot \overline{AN} \cdot \operatorname{sen}\alpha}{2} \\ \hat{MP'P} \simeq \hat{AMN} \quad \overline{AM} : \overline{AN} = MP' : PP' \end{cases}$$

$$\begin{cases} \overline{AN} = \dfrac{2 S_1}{AM \operatorname{sen}\alpha} \\ \overline{AM} : \overline{AN} = (AM - b) : a \end{cases} \qquad \begin{cases} AN = \dfrac{2 S_1}{AM \operatorname{sen}\alpha} \\ AM \cdot a = AN \cdot (AM - b) \end{cases}$$

$$\overline{AM} \cdot a = \frac{2S}{AM \operatorname{sen}\alpha} \cdot (\overline{AM} - b)$$

$$AM^2 \cdot a \cdot \operatorname{sen}\alpha = 2S \cdot \overline{AM} - 2S \cdot b \qquad \overline{AM} = ?$$

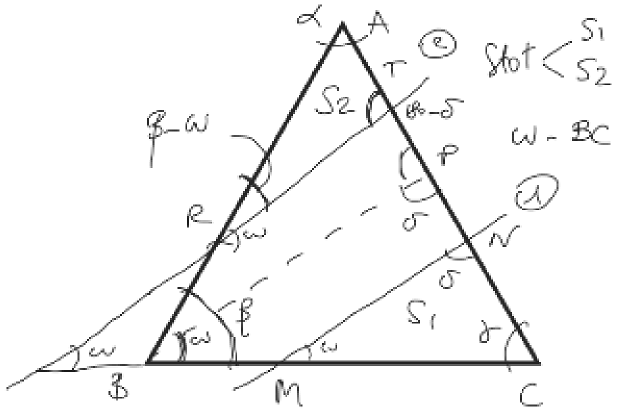
$$AM^2 \cdot a \cdot \operatorname{sen}\alpha - 2S \cdot \overline{AM} + 2S \cdot b = 0$$

$$\overline{AM} = \frac{2S \pm \sqrt{4S^2 - 8Sba \cdot \operatorname{sen}\alpha}}{\ldots} < \begin{matrix} \text{2 soluzioni reali} \\ \text{1 sol reale} \end{matrix}$$

$\overline{}$ $2a\,\text{sen}\,\alpha$ — no soluzione

Divisione di un triangolo con una dividente che formi con un lato un angolo assegnato

$$S_{B\hat{P}C} = \frac{\overline{BC}^2\,\text{sen}\,\omega\,\text{sen}\,\gamma}{2\,\text{sen}\,\delta} \gtreqless S_1 \begin{cases} > & (1) \\ < & (2) \end{cases}$$



$S_{tot} < \begin{matrix} S_1 \\ S_2 \end{matrix}$

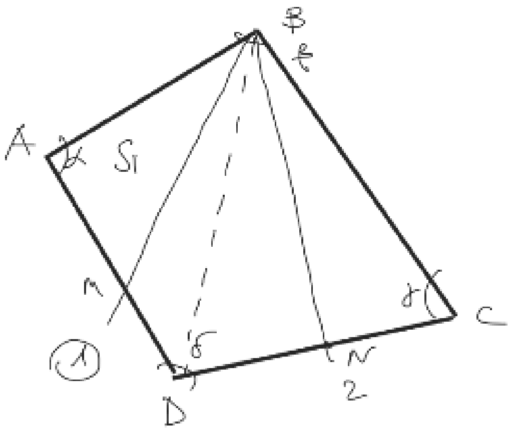
ω - BC

1) $$S_1 = S_{MNC} = \frac{\overline{CM}^2\,\text{sen}\,\omega\,\text{sen}\,\gamma}{2\,\text{sen}\,\delta}$$

$$\Rightarrow \overline{CM} = \sqrt{\frac{2S_1\,\text{sen}\,\delta}{\text{sen}\,\omega \cdot \text{sen}\,\gamma}}$$

2) $$\underline{\underline{S_{ATR} = S_2 = S_{tot} - S_1}} = \frac{\overline{AR}^2 \cdot \text{sen}\,\alpha \cdot \text{sen}(\beta-\omega)}{2\,\text{sen}(180-\delta)} \Rightarrow \overline{AR} = \sqrt{\frac{2S\,\text{sen}(180-\delta)}{\text{sen}\,\alpha\,\text{sen}(\beta-\omega)}}$$

Divisione di un quadrilatero con dividenti uscenti da un vertice



e.n $\begin{cases} \text{lati} \\ \text{angol.} \\ S_{tot} < S_1, S_2, \dots, S_n \\ \text{uscente } B \end{cases}$

$$S_{ABD} = \frac{\overline{AB}\,\overline{AD}\,\text{sen}\,\alpha}{2} \gtreqless S_1 \begin{cases} > & (1) \\ < & (2) \end{cases}$$

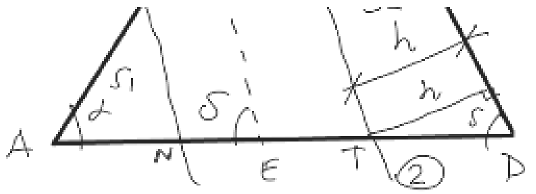
1) $$S_{ABM} = S_1 = \frac{\overline{AB} \cdot \overline{AM}\,\text{sen}\,\alpha}{2} \Rightarrow \overline{AM} = \frac{2S_1}{AB\,\text{sen}\,\alpha}$$

2) $$S_{BCN} = S_{tot} - S_1 = \frac{\overline{BC} \cdot \overline{CN}\,\text{sen}\,\gamma}{2} \Rightarrow \overline{CN} = \frac{2(S_{tot} - S_1)}{BC\,\text{sen}\,\gamma}$$

Divisione di un quadrilatero con dividenti parallele ad un lato

$S_{tot} < \begin{matrix} S_1 \\ S_2 \end{matrix}$

// CD



$$S_{ABE} = \frac{\overline{AB}^2 \cdot \operatorname{sen}\alpha \cdot \operatorname{sen}(\delta+\alpha)}{2 \operatorname{sen}\delta} \gtreqless S_1 \quad \begin{matrix} > \text{ ① TRIANGOLI} \\ < \text{ ② CDRT TRAPEZIO } CD // RT \end{matrix}$$

2a) Formula del trapezio

$$\begin{cases} S_2 = \dfrac{(\overline{CD} + \overline{RT})\,h}{2} \\ \overline{RT} = \overline{CD} - \overline{DK} + \overline{CH} = \overline{CD} - h \cdot \operatorname{ctg}\delta + h \operatorname{ctg}(200-\gamma) \end{cases}$$

$$2S_2 = CD \cdot h + CD\,h - h^2 \operatorname{ctg}\delta + h^2 \operatorname{ctg}\gamma_1 \qquad \text{II° grado}$$

$$h^2(\operatorname{ctg}\delta - \operatorname{ctg}\gamma_1) - 2\,\overline{CD} \cdot h + 2S_2 = 0$$

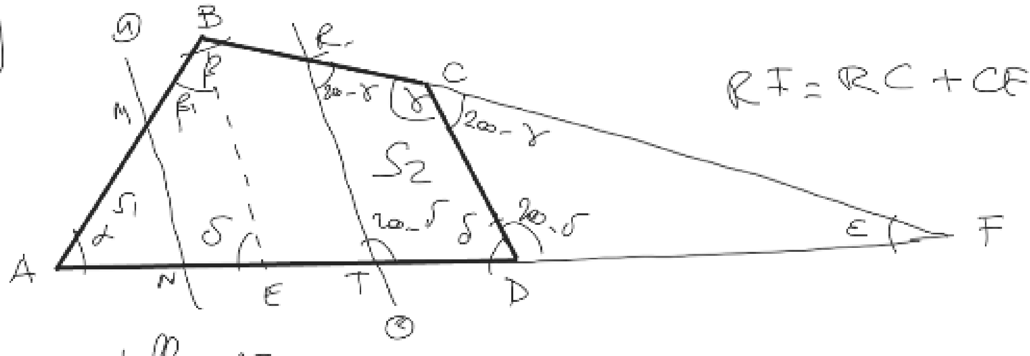
$$\Rightarrow h = \frac{2 \cdot \overline{CD} \pm \sqrt{4CD^2 - 8(\operatorname{ctg}\delta - \operatorname{ctg}\gamma_1) \cdot S_2}}{2(\operatorname{ctg}\delta - \operatorname{ctg}\gamma_1)} \qquad \text{soluzioni reali}$$

$$\overline{DK} = h \operatorname{ctg}\delta \qquad \overline{CH} = h \operatorname{ctg}\gamma_1$$

2b) $$S_{CDF} = \frac{\overline{CD}^2 \cdot \operatorname{sen}\gamma_1 \cdot \operatorname{sen}(200-\delta)}{2 \operatorname{sen}\varepsilon}$$

$$\varepsilon = 200 - (\gamma_1 + 200 - \delta)$$

2b)



$$RF = RC + CF$$

$\rightarrow \varepsilon$ per differenza

$\rightarrow$ risolvere CDF noti $\begin{cases} CD \\ 200-\gamma \\ 200-\delta \\ \varepsilon \end{cases}$ $\triangle$

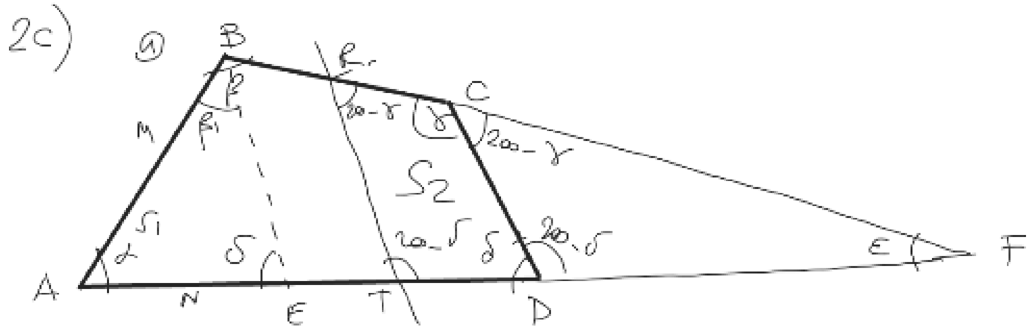
$$\rightarrow S_{CDF} = \frac{CD^2 \operatorname{sen}(200-\gamma) \operatorname{sen}(200-\delta)}{2 \operatorname{sen}\varepsilon}$$

$$S_{RTF} = S_{CDF} + S_2 = \frac{\overline{RF}^2 \cdot \operatorname{sen}\varepsilon \cdot \operatorname{sen}(200-\gamma)}{2 \operatorname{sen}(200-\delta)}$$

$$\Rightarrow \overline{RF} = \sqrt{\frac{2 \cdot S_{RTF} \cdot \operatorname{sen}(200-\delta)}{\operatorname{sen}\varepsilon \cdot \operatorname{sen}(200-\gamma)}}$$

$\overline{RC} = RF - CE$; $\overline{TD} = TF - DF$

2c)

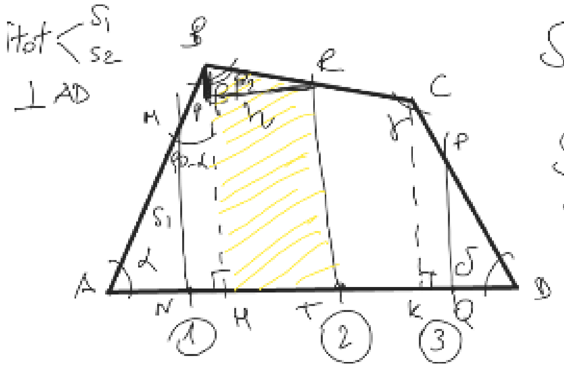


$\overset{\triangle}{RTF}$ simile $\overset{\triangle}{CDF}$

$S_{CDF} : S_{RTF} = \overline{CF}^2 : \overline{RF}^2$

$\overline{RF} = \sqrt{\frac{\overline{CF}^2 (S_2 + S_{CDF})}{S_{CDF}}}$ ⇐

Divisione di un quadrilatero con dividenti perpendicolari ad un lato



$S_{ABH} = \frac{\overline{BH}\cdot\overline{AH}}{2} = \frac{\overline{AB}\sin\alpha \cdot \overline{AB}\cos\alpha}{2} = \frac{\overline{AB}^2\sin\alpha\cos\alpha}{2}$

$S_{ABH} > S_1$ (1) MN

$S_{ABH} < S_1$ (2) (3) ⇒ $S_{CKD} = \frac{\overline{CD}^2\sin\delta\cos\delta}{2}$

$S_{CDK} > S_2$ (3)

$S_{CDK} < S_2$ (2)

(1) e (3) $\overset{\triangle}{AMN}$ o $\overset{\triangle}{PQD}$ ⇐ TRIANGOLI | (2) QUADRILATERO = TRAPEZIO

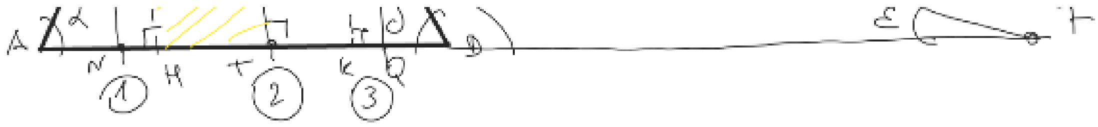
2a) Problema del trapezio

$S_{BRTH} = S_1 - S_{ABH} = \frac{\overline{BH}+\overline{RT}}{2} h$; $\overline{RT} = \overline{BH} - \overline{BP} = \overline{BH} - h\,\mathrm{ctg}\beta_2$

$2S_{BRTH} = BH \cdot h + \overline{BH} \cdot h - h^2\mathrm{ctg}\,\beta^2$

$h^2\mathrm{ctg}\beta - 2BH\cdot h + 2S_{BRTH} = 0$ h = eq. II° grado soluzioni reali

2b) Prolungamento



CDF risolvo con il T. dei seni

S_{CDF}

$S_{RTF} = S_2 + S_{CDF}$ $\begin{cases} \frac{\overline{RT} \cdot \overline{TF}}{2} = \frac{\overline{TF}^2 \cdot tg\,\varepsilon}{2} \\ RT = TF \cdot tg\,\varepsilon \end{cases}$ $\Rightarrow \overline{TF} = \sqrt{\frac{2(S_2 - S_{CDF})}{tg\,\varepsilon}}$

$\overline{DT} = \overline{TF} - \overline{DF}$